PART - C

$10 \times 10 = 100$

Note : (i) Answer any ten questions.

(ii) Question No. 70 is compulsory and choose any nine questions from the remaining.

56. Solve the following non-homogeneous system of linear equations by determinant method:

$$\frac{1}{x}+\frac{2}{y}-\frac{1}{z}=1;\ \frac{2}{x}+\frac{4}{y}+\frac{1}{z}=5;\ \frac{3}{x}-\frac{2}{y}-\frac{2}{z}=0$$

57. Discuss the solutions of the system of equations for all values of λ .

$x+y+z=2,\quad 2x+y-2z=2,\quad \lambda x+y+4z=2$

58. Prove that $\cos(A-B)=\cos A\cos B+\sin A\sin B$.

59. Find the vector and Cartesian equation to the plane through the point (-1,-2,1) and perpendicular to the planes $x+2y+4z+7=0$ and $2x-y+3z+3=0$.

60. If α and β are the roots of the equation $x^2-2px+(p^2+q^2)=0$ and $\tan\theta=\frac{q}{y+p}$ show that

$$\frac{(y+\alpha)^n-(y+\beta)^n}{\alpha-\beta}=q^{n-1}\frac{\sin n\theta}{\sin^n\theta};\ n\in N.$$

61. Solve: $x^4-x^3+x2-x+1=0$.

62. Find the axis, vertex, focus, directrix, equation of the latus rectum, length of the latus rectum of the parabola $x^2-2x+8y+17=0$ and hence draw the diagram.

63. The ceiling in a hallway 20 ft wide is in the shape of a semi ellipse and 18 ft high at the centre. Find the height of the ceiling 4 feet from either wall if the height of the side walls is 12 ft.

64. Show that the line $x-y+4=0$ is a tangent to the ellipse $x^2+3y^2=12$. Find the co-ordinates of the point of contact.

65. Find the equation of the rectangular hyperbola which has for one of its asymptotes the line $x+2y-5=0$ and passes through the points (6, 0) and (-3, 0).

66. Prove that the sum of the intercepts on the co-ordinate axes of any tangent to the curve $x=a\cos^4\theta$, $y=a\sin^4\theta$, $0\le\theta\le\frac{\pi}{2}$ is equal to 'a'.

67. Show that the volume of the largest right circular cone that can be inscribed in a sphere of radius 'a' is $\frac{8}{27}$ (volume of the sphere).

68. Trace the curve $y=x^3+1$.

69. If $w=u^2e^v$ where $u=\frac{x}{y}$ and $v=y\log x$, find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$.

70. a) If $\vec{a}=2\vec{i}+3\vec{j}-\vec{k}$, $\vec{b}=-2\vec{i}+5\vec{k}$, $\vec{c}=\vec{j}-3\vec{k}$ prove that $\vec{a}\ X\ (\vec{b}\ X\ \vec{c})=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$.

(OR) (b) Find the points of inflection and determine the intervals of convexity and concavity of the Gaussion curve $y=e^{-x2}$.